

# Hypernuclear matter in strong magnetic field

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## Abstract

Compact stars with strong magnetic fields (magnetars) have been observationally determined to have surface magnetic fields of order of  $10^{14} - 10^{15}$  G, the implied internal field strength being several orders larger. We study the equation of state and composition of dense hypernuclear matter in strong magnetic fields in a range expected in the interiors of magnetars. Within the non-linear Boguta-Bodmer-Walecka model we find that the magnetic field has sizable influence on the properties of matter for central magnetic field  $B \geq 10^{17}$  G, in particular the matter properties become anisotropic. Moreover, for the central fields  $B \geq 10^{18}$  G, the magnetized hypernuclear matter shows instability, which is signalled by the negative sign of the derivative of the pressure parallel to the field with respect to the density, and leads to vanishing parallel pressure at the critical value  $B_{\text{cr}} \simeq 10^{19}$  G. This limits the range of admissible homogeneously distributed fields in magnetars to fields below the critical value  $B_{\text{cr}}$ .

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## 1. Introduction

Anomalous X-ray pulsars and soft  $\gamma$ -ray repeaters are observationally identified with highly magnetized neutron stars with surface magnetic field  $\sim 10^{14} - 10^{15}$  G [1–3]. This class of compact stars has been conjectured theoretically as “magnetars” [4–7], *i.e.*, neutron stars which possess magnetic fields that are by several orders of magnitude larger than the canonical surface dipole magnetic fields  $B \sim 10^{12}$ - $10^{13}$  G of the bulk of pulsar population, which are commonly deduced from the magnetic-dipole radiation model of pulsar spin-down in combination with measured spin and spin-down rates. The integral properties of magnetars, *i.e.*, mass, radius, moment of inertia, *etc* will depend sensitively on the equation of state of matter in strong magnetic fields, if the central fields are sufficiently strong. Furthermore, other processes, such as the cooling and the magnetic field evolution will sensitively depend on the composition of matter in strong magnetic fields. Fermionic matter

in strong magnetic field experiences two well-known quantum mechanical phenomena: the Pauli paramagnetism and the Landau diamagnetism. The first is due to the interaction of the spin of the fermion with the magnetic field and, therefore, is relevant for both charged and uncharged fermions. The second phenomenon is relevant only for charged particles, and is particularly strong for light particles, which in the case of compact stars are the leptons.

Neutron star sequences may feature massive objects with masses  $M \sim 2M_{\odot}$ . These massive compact stars are likely to develop cores which are composed of matter that differs from the ordinary nuclear matter composed of only neutrons and protons. One possibility is that heavy baryons (hyperons) will appear once the Fermi energy of neutrons becomes of the order of their rest mass. Although the hyperons were considered even before the discovery of pulsars and their identification with the neutron stars [8], their presence in the cores of neutron stars is still quite uncertain, for different theoretical models predict quite different outcomes for the maximum mass of hypernuclear compact stars, often in contradiction with the known empirical data on pulsar masses. The models that are based on relativistic density functional methods [9–12] predict masses that are not much larger than the canonical mass of a neutron star which clearly contradicts modern observations. Masses of the order of  $\leq 1.8M_{\odot}$  were obtained in non-relativistic phenomenological models [13, 14], while microscopic models based on hyperon-nucleon potentials, which include the repulsive three-body forces, predict low maximal masses for hypernuclear stars [15, 16]. The avenues for reconciliation of the large pulsar mass and the hyperonization (and more generally strangeness) of dense matter have been explored recently in Refs [17–28].

The influence of strong magnetic fields on the highly dense electron gas in the context of neutron star matter was studied in Refs. [29–33]. The strong magnetic field effects on dense nuclear matter ( $n, p, e$  system) have been studied previously in Refs. [34–40]. The structure of strongly magnetized neutron star branch of compact objects was studied for different field configurations (toroidal, poloidal, etc) in Refs. [41–43]. In the case of the adjacent branch of white dwarfs, strong magnetic fields were found to lead to highly super-Chandrasekhar mass ( $M \sim 2.3 - 2.6M_{\odot}$ ) white dwarfs [44–46], which can be related to the observed features of a number of peculiar Type Ia supernovae [47].

It has been known for some time that the magnetic field can affect the hydrostatic equilibrium of compact stars and may render large fields configurations unstable. In the simplest form it can be formulated for uniform self-gravitating fluids [48]. In the case of neutron stars the Chandrasekhar-Fermi limiting field strength is  $\sim 10^{18}$  G [49]. Fully relativistic calculations confirm the simple Newtonian estimates [41–43]. Instabilities related to the anisotropy of the pressure were also discussed in the literature. These types of instabilities may arise due to the change of the sign of the derivative of either the transverse pressure or the parallel pressure with respect to the density, which leads eventually to vanishing of the respective pressure. These instabilities have been discussed for electron gas [50] and strange quark matter [51–53] due to the vanishing of transverse pressure and for magnetized fermionic systems [54] and quark matter [55, 56] due to parallel pressure (but see [57, 58]).

The studies of magnetized dense matter were mainly carried out in the limit of uniform field distribution, some notable exceptions are Refs. [35, 54, 59]. The processes of supernova

collapse will leave behind a strongly non-uniform field distribution of frozen-in fields. Any dynamo mechanism generating fields will carry the imprint of inhomogeneous density profile in the star. Thus, more realistic treatment of the equation of state of matter requires inclusion of some physically motivated field profiles of the stars such as magnetars. (We note that *local* equation of state is not affected by the density profile of the field, and therefore depends only on the local value of the magnetic field strength and induction). Thus, while uniform magnetic field distribution gives a crude order of magnitude estimates, it requires refinements, because the density profiles of neutron stars are not uniform. Consequently, the local magneto-static equilibrium will be achieved for different field strength.

The motivation of this paper is to carry out a study which includes the following aspects of the physics of strong magnetic field discussed above: (i) we allow for radial profile of the magnetic field, which assumes a high-field central value which decays as the field stretches towards the surface. In this way, *we extend the existing studies of hypernuclear matter to the case of non-uniform fields*. (ii) We carefully analyze the different components of the field and show, in our case study of hypernuclear matter, that *the parallel pressure shows the instability*. Thus, our work combines and unifies the ideas of the non-homogenous distribution of magnetic fields in magnetars and possible instabilities in a new context of nuclear and hypernuclear matter. We quantify these effects by studying the dependence of the equation of state, onset of instability, and composition of matter by varying our parameterization of the field profiles.

This paper is organized as follows. In Sec. 2 we discuss our model for hypernuclear matter in strong magnetic fields within a relativistic density functional theory. We present our numerical results in Sec. 3. Section 4 contains a summary of our results.

## 2. Model

As discussed in the introduction, the treatment of hypernuclear matter within nuclear models which are based on different principles has lead to array of results, some of which are inconsistent with measurements of pulsar masses. Relativistic models of hypernuclear matter are flexible enough to be adjusted to the current phenomenology, therefore they provide a suitable framework for extensions which incorporate the influence of strong magnetic fields. In particular, we consider the non-linear Boguta-Bodmer-Walecka model which is based on relativistic density functional theory [9, 60–64]. This model is able to describe ground state phenomenology of nuclear matter and nuclei (see, e.g., [65, 66]). We anticipate that the main conclusions and results of the present study will not change qualitatively if a different model of dense hypernuclear matter is chosen.

The Lagrangian density of hypernuclear matter in a static magnetic field (hereafter also  $B$ -field) is given by

$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_f, \quad (1)$$

where the matter part of the Lagrangian is given by

$$\mathcal{L}_m = \sum_B \bar{\psi}_B (i\gamma_\mu D^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - g_{\rho B}\gamma_\mu\boldsymbol{\tau}_B \cdot \boldsymbol{\rho}^\mu) \psi_B$$

$$\begin{aligned}
& +\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) \\
& -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu \\
& + \sum_{l=e,\mu} \bar{\psi}_l (i\gamma_\mu D^\mu - m_l) \psi_l,
\end{aligned} \tag{2}$$

where the index  $B$  labels baryons present in the matter,  $\psi_B, \psi_l, \sigma, \omega$  and  $\boldsymbol{\rho}$  are fields of baryons, leptons,  $\sigma$ -mesons,  $\omega$ -mesons and  $\rho$ -mesons, with masses  $m_B, m_l, m_\sigma, m_\omega$  and  $m_\rho$  respectively;  $g_{\sigma B}, g_{\omega B}$  and  $g_{\rho B}$  are coupling constants for interactions of  $\sigma, \omega$  and  $\rho$  mesons respectively with the baryon  $B$ . The scalar self interaction term in the matter Lagrangian is [9, 61–64]

$$U(\sigma) = \frac{1}{3} g_1 m_N (g_{\sigma N} \sigma)^3 + \frac{1}{4} g_2 (g_{\sigma N} \sigma)^4, \tag{3}$$

where  $g_1$  and  $g_2$  are constants which parameterize the shape of the potential and

$$\omega_{\mu\nu} = \partial_\nu\omega_\mu - \partial_\mu\omega_\nu, \tag{4}$$

$$\boldsymbol{\rho}_{\mu\nu} = \partial_\nu\boldsymbol{\rho}_\mu - \partial_\mu\boldsymbol{\rho}_\nu, \tag{5}$$

$$D^\mu = \partial^\mu + ieQA^\mu. \tag{6}$$

We choose the field vector potential gauge as  $A^\mu \equiv (0, -yB, 0, 0)$ , where  $B$  is the magnitude of magnetic field and  $eQ$  the charge of the particle,  $e$  the (positive) unit of charge. For this particular gauge choice the magnetic field is along the  $z$  axis of Cartesian coordinate system, *i.e.*,  $\mathbf{B} = B\hat{z}$ . In the mean field approximation, the baryons acquire the effective masses

$$m_B^* = m_B - g_{\sigma B}\sigma, \tag{7}$$

where  $\sigma$  is given by its ground state expectation value

$$\langle \sigma \rangle = \sigma = \frac{1}{m_\sigma^2} \left( \sum_B g_{\sigma B} n_S^{(B)} - \frac{\partial U}{\partial \sigma} \right), \tag{8}$$

with the scalar density

$$n_S^{(B)} = \frac{2}{(2\pi)^3} \int_0^{p_F^{(B)}} \frac{m_B^*}{\sqrt{p_B^2 + m_B^{*2}}} d^3p_B, \tag{9}$$

where  $p_B$  is the momentum and  $p_F^{(B)}$  the Fermi momentum of the baryon  $B$ . The electromagnetic field Lagrangian density is given by

$$\mathcal{L}_f = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu}, \tag{10}$$

where  $F^{\mu\nu}$  is the electromagnetic field tensor.

The total energy density and pressure of the system can be obtained by computing the energy-momentum tensor from the Lagrangian density (1). The result is

$$T^{\mu\nu} = T_m^{\mu\nu} + T_f^{\mu\nu}, \quad (11)$$

where the matter part is given by

$$T_m^{\mu\nu} = \varepsilon_m u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu) + \frac{1}{2}(M^{\mu\lambda} F_\lambda^\nu + M^{\nu\lambda} F_\lambda^\mu), \quad (12)$$

with  $\varepsilon_m$  being the matter energy density,  $P$  - the thermodynamic pressure,  $M^{\mu\nu}$  - the magnetization tensor. The field part of the energy-momentum tensor is given by

$$T_f^{\mu\nu} = -\frac{1}{4\pi} F^{\mu\lambda} F_\lambda^\nu + \frac{1}{16\pi} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}. \quad (13)$$

In the following we will neglect the electric field, as there are no macroscopic charges in the bulk matter. Therefore, Eqs. (12) and (13) reduce, respectively, to [52, 67]

$$T_m^{\mu\nu} = \varepsilon_m u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu) + MB \left( g^{\mu\nu} - u^\mu u^\nu + \frac{B^\mu B^\nu}{B^2} \right), \quad (14)$$

$$T_f^{\mu\nu} = \frac{B^2}{4\pi} \left( u^\mu u^\nu - \frac{1}{2} g^{\mu\nu} \right) - \frac{B^\mu B^\nu}{4\pi}, \quad (15)$$

with  $B^\mu B_\mu = -B^2$  and  $M$  being the magnetization per unit volume.

In the presence of the magnetic field, the motion of the charged particles is Landau quantized in the direction perpendicular to the magnetic field. We choose the coordinate axes in such a way that  $B$  is along  $z$ -axis. Then, the single particle energy of any charged particle at  $n$ -th Landau level is given by

$$E_n = \sqrt{p_z^2 + m^2 + 2ne|Q|B}, \quad (16)$$

where  $m$  is the mass of the particle,  $p_z$  is the component of momentum along  $z$  direction. The Landau levels in Eq. (16) assume integer values  $n = 0, 1, 2, \dots$  for spin-up states and  $n = 1, 2, 3, \dots$  for spin-down states for positively charged particles. For negatively charged particles  $n$  takes on values  $n = 0, 1, 2, \dots$  for the spin-down states and  $n = 1, 2, 3, \dots$  for spin-up states. The zero-temperature number density of charged baryons and leptons is given by

$$n_C = \frac{e|Q|B}{2\pi^2} \sum_{n=0}^{n_{max}} (2 - \delta_{n,0}) \sqrt{p_F^2 - 2ne|Q|B}, \quad (17)$$

with

$$n_{max} = \text{Int} \left( \frac{p_F^2}{2e|Q|B} \right), \quad (18)$$

where  $p_F$  is the Fermi momentum. At zero temperature the number density of neutral baryons is expressed via their Fermi momentum as

$$n_N = \frac{p_F^3}{3\pi^2}. \quad (19)$$

The chemical potentials of baryons are given by  $\mu_B = \mu_B^* + \omega^0 g_{\omega B} + \rho_3^0 I_3^{(B)} g_{\rho B}$ , where

$$\mu_B^* = \sqrt{p_F^{(B)2} + m_B^{*2}}, \quad (20)$$

with  $m_B^*$  being the effective mass of the baryon  $B$  in the mean field approximation. We will further need the quantities

$$\omega^0 = \frac{1}{m_\omega^2} \sum_B g_{\omega B} n_B, \quad (21)$$

$$\rho_3^0 = \frac{1}{m_\rho^2} \sum_B g_{\rho B} I_3^{(B)} n_B, \quad (22)$$

which are the ground state expectation values of  $\omega$  and  $\rho_3$  fields with  $I_3^{(B)}$  being the isospin projection for the baryon  $B$ . The chemical potentials of leptons are given by

$$\mu_l = \sqrt{p_F^{(l)2} + m_l^2}, \quad (23)$$

where  $p_F^{(l)}$  and  $m_l$  are the Fermi momenta and masses of leptons, respectively. Now we are in the position to write down the matter energy density defined by the matter Lagrangian density,  $\mathcal{L}_m$ , given by Eq. (2)

$$\begin{aligned} \varepsilon_m &= \frac{1}{8\pi^2} \sum_B \left( 2p_F^{(B)} \mu_B^{*3} - p_F^{(B)} m_B^{*2} \mu_B^* - m_B^{*4} \ln \frac{p_F^{(B)} + \mu_B^*}{m_B^*} \right) \\ &+ \frac{e|Q|B}{(2\pi)^2} \sum_{B'} \sum_{n=0}^{n_{max}} (2 - \delta_{n,0}) \left[ p_{B'}(n) \mu_{B'}^* + (m_{B'}^{*2} + 2ne|Q|B) \ln \left( \frac{p_{B'}(n) + \mu_{B'}^*}{\sqrt{m_{B'}^{*2} + 2ne|Q|B}} \right) \right] \\ &+ \frac{e|Q|B}{(2\pi)^2} \sum_{l=e,\mu} \sum_{n=0}^{n_{max}} (2 - \delta_{n,0}) \left[ p_l(n) \mu_l + (m_l^2 + 2ne|Q|B) \ln \left( \frac{p_l(n) + \mu_l}{\sqrt{m_l^2 + 2ne|Q|B}} \right) \right] \\ &+ \frac{1}{2} m_\sigma^2 \sigma^2 + U(\sigma) + \frac{1}{2} m_\omega^2 \omega^0{}^2 + \frac{1}{2} m_\rho^2 \rho_3^0{}^2, \end{aligned} \quad (24)$$

where the sums over  $B$  and  $B'$  include the uncharged and charged baryons, respectively, and the following short-hand notation has been used

$$p(n) = \sqrt{p_F^2 - 2ne|Q|B}. \quad (25)$$

The matter pressure can be easily constructed from Eq. (24) using the thermodynamic relation

$$P = \sum_B \mu_B n_B + \sum_l \mu_l n_l - \varepsilon_m. \quad (26)$$

We now account for the fact that the matter in compact stars is charge neutral and in beta equilibrium. The first requirement relates the partial densities of charged particles according to

$$n_p + n_{\Sigma^+} = n_{\Sigma^-} + n_{\Xi^-} + n_e + n_\mu. \quad (27)$$

Equilibrium with respect to weak interactions further implies the following relations

$$\begin{aligned} \mu_p &= \mu_n - \mu_e, & \mu_\mu &= \mu_e, & \mu_\Lambda &= \mu_n, & \mu_{\Sigma^-} &= \mu_n + \mu_e, \\ \mu_{\Sigma^0} &= \mu_n, & \mu_{\Sigma^+} &= \mu_p, & \mu_{\Xi^-} &= \mu_n + \mu_e, & \mu_{\Xi^0} &= \mu_n. \end{aligned} \quad (28)$$

These conditions are normalized such that the total number of baryons is reproduced, *i.e.*,

$$n_b = \sum_B n_B. \quad (29)$$

The solutions of the field equations at any given baryon density  $n_B$  and zero temperature are found in the mean-field approximation under the constraints (27), (28), and (29). Subsequently, the energy density and the thermodynamic pressure of the matter are obtained using Eqs. (24) and (26).

In the rest frame of the hadronic fluid, with  $B$  field along the  $z$  axis, the matter and field parts of the energy-stress tensor are given, respectively, by

$$T_m^{\mu\nu} = \begin{pmatrix} \varepsilon_m & 0 & 0 & 0 \\ 0 & P - MB & 0 & 0 \\ 0 & 0 & P - MB & 0 \\ 0 & 0 & 0 & P \end{pmatrix}, \quad (30)$$

$$T_f^{\mu\nu} = \frac{B^2}{8\pi} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (31)$$

The total energy density of the system is given by the sum of the matter and field contributions

$$\varepsilon = \varepsilon_m + \frac{B^2}{8\pi}. \quad (32)$$

It is seen from Eqs. (30) and (31) that the the pressure in the perpendicular direction to the magnetic field is

$$P_\perp = P - MB + \frac{B^2}{8\pi}, \quad (33)$$

and the pressure in the direction parallel to the magnetic field is given by

$$P_\parallel = P - \frac{B^2}{8\pi}. \quad (34)$$

The couplings in the hypernuclear Lagrangian are fixed as follows. In the nucleonic sector the nucleon-meson coupling constants are chosen according to Refs. [9, 62–64] and reproduce

the bulk properties of nuclear matter (the binding energy  $E/B = -16.3$  MeV, the saturation density  $n_0 = 0.153 \text{ fm}^{-3}$ , the asymmetry energy coefficient  $a_{asy} = 32.5$  MeV, the incompressibility  $K = 240$  MeV and the effective nucleon mass at the saturation  $m^*/m = 0.8$ ). In the hyperonic sector, the hyperon- $\omega$  coupling constants is derived from the  $SU(6)$  symmetry of the quark model [68–70]. To infer the hyperon- $\sigma$  coupling constants we write the potential depth of a hyperon  $Y$  in nuclear matter at saturation as

$$U_Y = -g_{\sigma Y}\sigma + g_{\omega Y}\omega_0, \quad (35)$$

and assign certain value to the potential  $U_Y$ , which then fixes the value  $g_{\sigma Y}$ . We take the potential depth for  $\Lambda$ -hyperon  $U_\Lambda = -30$  MeV, as obtained from the analysis of  $\Lambda$ -hypernuclei [68, 71]. For  $\Xi$ -hyperon we assign  $U_\Xi = -18$  MeV according to experimental data of Refs. [71, 72]. According to the recent  $\Sigma$ -hypernuclei data the potential for  $\Sigma$  hyperons is repulsive [73] and we adopt the value  $U_\Sigma = 30$  MeV.

To model the density profile of the magnetic field we adopt the following formula [59]

$$B\left(\frac{n_b}{n_0}\right) = B_s + B_c \left\{ 1 - \exp\left[-\beta\left(\frac{n_b}{n_0}\right)^\gamma\right] \right\}. \quad (36)$$

As discussed in the introduction the present formula allows us to model the realistic physical situations where the magnetic field of the star is non-uniform which is a realistic situation. The functional dependence of the profile (36) on density is constructed such as to take into account the fact that the magnetic field in the cores of compact stars may be higher than that at the surface. The parameters  $\beta$  and  $\gamma$  control the relaxation from the central value  $B_c$  to the asymptotic value at the surface  $B_s$ . Obviously, the parameter  $\beta$  controls the amount of field decay at the saturation density, whereas the parameter  $\gamma$  controls the width of the transition. Observationally inferred value of the surface magnetic field of magnetars is of the order of  $10^{15}$  G. It is plausible to assume that the central field could be in the range of  $10^{17} - 10^{18}$  G. Therefore, we will consider a number of different values of  $B_c$ , while keeping  $B_s = 10^{15}$  G. We have verified that the results are insensitive to the precise value of the surface field  $B_s$ .

### 3. Results and discussion

Our model of the equation of state of hypernuclear matter in strong magnetic field is parameterized in terms of four physical quantities: the boundary values of the field at the center and the surface, the amount of its decline from the center to the saturation density (which is close to the crust-core interface) and the steepness of the transition between the two asymptotic values of the  $B$ -field. The latter two are described by the parameters  $\beta$  and  $\gamma$ , respectively. While the surface  $B$ -field has been fixed at the value of  $10^{15}$  G, we have explored a range of central magnetic fields; it turns out that the equation of state is unaffected by the magnetic fields below the value  $B_c \leq 10^{18}$  G.

Figure 1 shows the equation of state of hypernuclear matter in strong magnetic field. The left panel shows the dependence of pressure on the density for a fixed central  $B$ -field



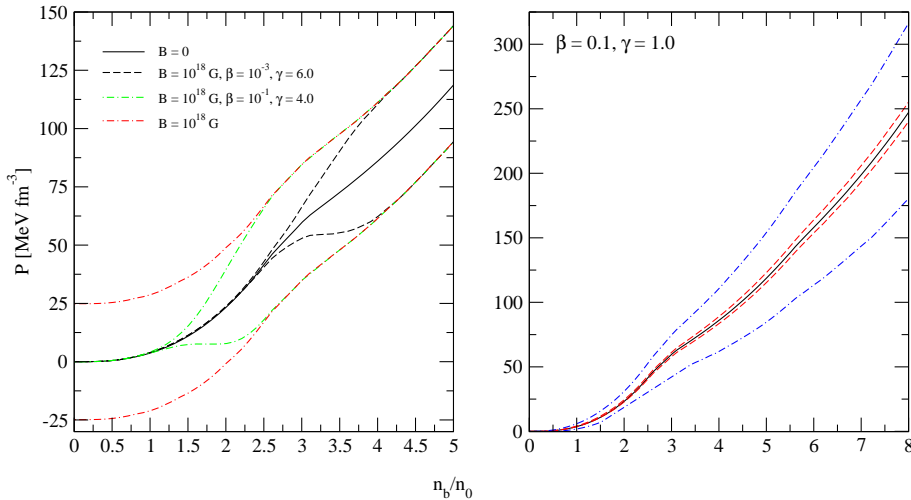


Figure 1: *Left panel:* Variation of total pressure as a function of normalized baryon number density for fixed magnetic fields  $B_c = 0$  and  $B_c = 10^{18}$  G and several field profiles,  $\beta = 10^{-3}$ ,  $\gamma = 6$  (dashed lines),  $\beta = 10^{-1}$ ,  $\gamma = 4$  (dashed-double-dotted lines), and  $\beta \rightarrow \infty$ , *i.e.*,  $B_c = \text{constant}$  (dashed-dotted lines). For each pair of curves the upper branch is for  $P_{\perp}$  and the lower branch for  $P_{\parallel}$ . *Right panel:* Same as in the left panel, but for fixed  $\beta = 10^{-1}$ ,  $\gamma = 1$  and for two values of  $B$ -field,  $B_c = 10^{18}$  G (dashed lines) and  $B_c = 3 \times 10^{18}$  G (dashed-dotted lines). The solid line corresponds to the case  $B_c = 0$ .

and various field profiles. For non-zero magnetic field the pressure splits into the parallel and transverse components, which is evident from Eqs. (33) and (34), and is due to the gauge field contribution. The case with  $\beta = 10^{-1}$  acquires a larger field at the saturation density  $n = n_0$ , therefore the transition to the asymptotic value of  $B_c$  occurs earlier than in the case of  $\beta = 10^{-3}$ , both cases having approximately the same width of transition (see Fig. 2). It is clearly seen that the low-density behavior of the equation of state with constant  $B$ -field implies unrealistically large anisotropy of magnetic field up to the surface of the star, which is inconsistent with the inferred surface magnetic field of magnetars  $B_s \sim 10^{15}$  G. The right panel of Fig. 1 shows the equation of state in case of fixed density profile  $\beta = 10^{-1}$  and  $\gamma = 1$ , but for various central magnetic fields. The main effect seen in the figure is the splitting of the parallel and transverse pressure as the field becomes larger, this effect being sizable for fields above  $10^{18}$  G. Furthermore, we see that the increase of  $B_c$  causes the profile for  $P_{\parallel}$  to become softer; the fact that  $P_{\parallel} < P_{\perp}$  is due to negative contribution of the field pressure to  $P_{\parallel}$ .

From the left panel of Fig. 1 we observe that there is an onset of matter instability for chosen combinations of  $\beta$  and  $\gamma$  (corresponding to different magnetic field profiles) for fixed central field  $B_c$ . This instability is induced in the  $P_{\parallel}$  component of the pressure, which decreases with the density, because its field profile implies an increase of field strength.

To study the onset of instability in a more systematic manner, consider the dependence of  $P_{\parallel}$  on density for two central values of magnetic field  $B_c = 10^{18}$  G and  $B_c = 3 \times 10^{18}$  G and several sets of  $\gamma$  and  $\beta$  parameters. This is shown in the upper and lower panels of Fig. 2. It is seen that for a given value of  $\beta$  the corresponding EoS becomes softer as the

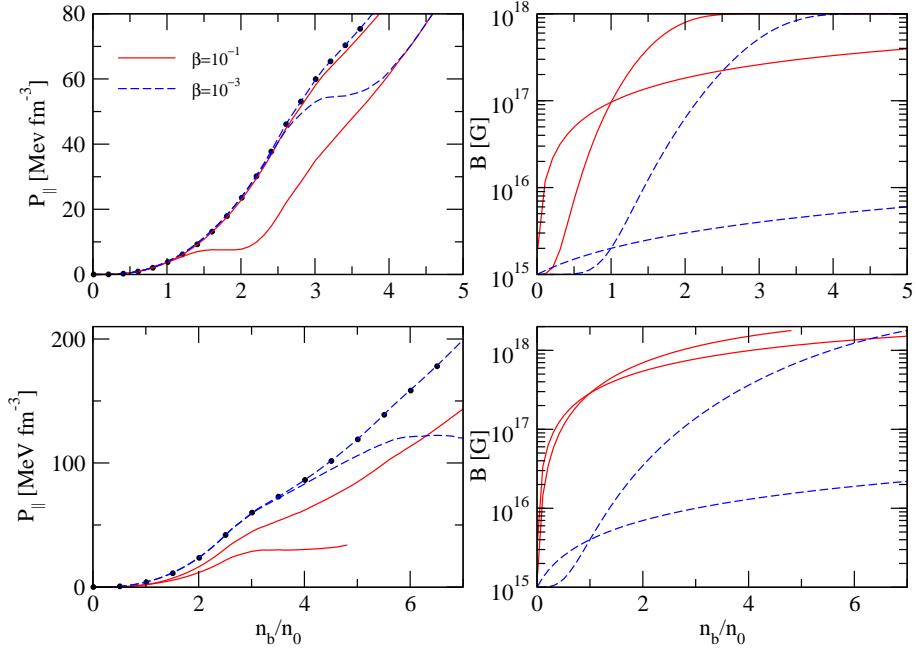


Figure 2: *Upper panels:* Dependence of  $P_{\parallel}$  and  $B$  on the normalized baryon number density for different magnetic field profiles and  $B_c = 10^{18}$  G. The dots show the reference case  $B_c = 0$ . The solid and dashed lines correspond to  $\beta = 0.1$  and  $0.001$ , respectively. For each  $\beta$  we choose a pair of  $\gamma$ 's; in the first case we have  $\gamma = 1$  and  $\gamma = 4$ , whereas in the second case  $\gamma = 1$  and  $\gamma = 6$ . The first  $\gamma$  value for each  $\beta$  corresponds to a less steeply rising curve. *Lower panels:* Same as in the upper panel but for  $B_c = 3 \times 10^{18}$  G and  $\gamma = 1$  and  $\gamma = 1.4$  for  $\beta = 0.1$  and  $\gamma = 1$  and  $\gamma = 3.5$  for  $\beta = 10^{-3}$ . The specific values of  $\gamma$ 's were chosen to demonstrate the onset of instability.

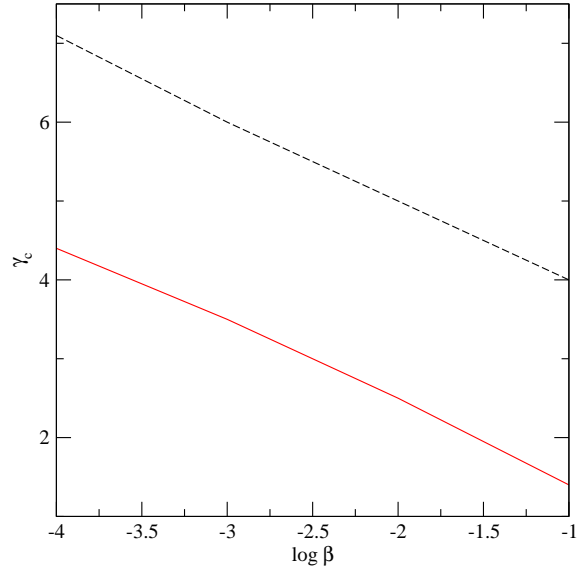


Figure 3: Dependence of  $\gamma_c$  - the critical value of  $\gamma$  parameter - on the logarithm of  $\beta$  parameter. The solid line corresponds to the field  $B_c = 3 \times 10^{18}$  G, the dashed line - to  $B_c = 10^{18}$  G

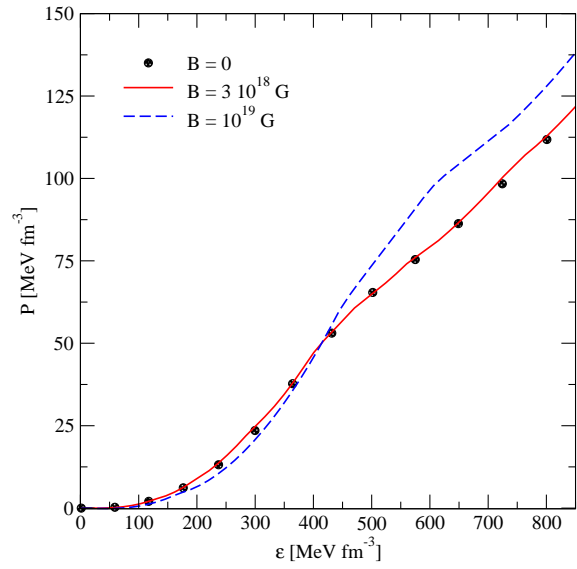


Figure 4: Dependence of matter pressure on normalized baryon number density without the  $B^2/8\pi$  term, which demonstrates the effect of Landau quantization.

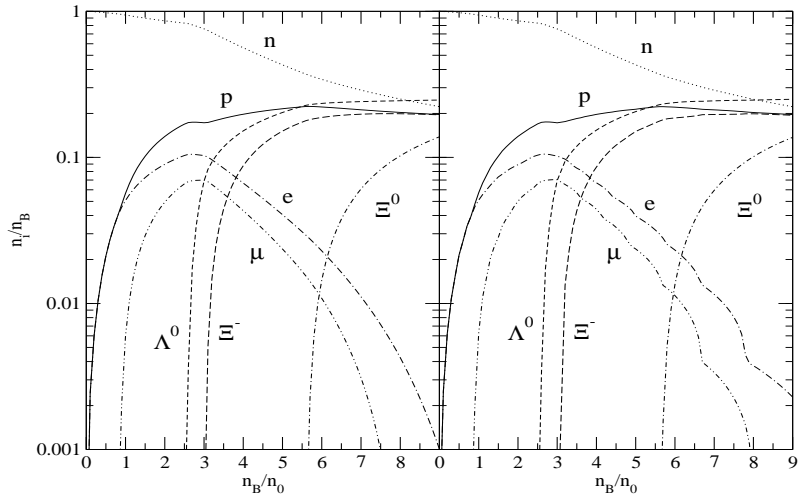


Figure 5: Abundances of different species as functions of normalized baryon number density without (left panel) and with (right panel) magnetic field  $B = 2 \times 10^{18}$  G. The field profile parameters are  $\beta = 10^{-1}$ ,  $\gamma = 1$ , i.e., the field variations across the density profile of the star is as illustrated in Fig. 2.

$\gamma$  is increased. Consequently, beyond a certain critical value of  $\gamma$  and in a certain density regime  $P_{\parallel}$  ceases to increase and subsequently decreases with the further increase in  $n_b$ . This implies that matter becomes unstable above that value of density for that particular  $B_c$  and magnetic field profile. This is evident from Fig. 2, where also the magnetic field profiles corresponding to the onset of instability are shown. For comparison we also show results for each  $\beta$  with the minimum value of  $\gamma$  taken to be 1. Note that the maximum value of  $\gamma$  is taken such that  $P_{\parallel}$  forms a plateau as a function  $n_b$ . Furthermore, it is evident that with the decrease of  $\beta$ , the instability occurs at larger values of  $\gamma$  and  $n_b$ , which is also evident from Fig 2.

The instability arises due to the negative contribution from the field energy density (pressure) to the pressures of magnetized baryons and leptons in the direction to the magnetic field, which is evident from Eq. (34). Since for any particular  $B_c$  and magnetic profile, the field strength increases with the increase of  $n_b$ , more negative contribution is added to  $P_{\parallel}$  with the increase of  $n_b$ . Consequently, at a certain density,  $P_{\parallel}$  ceases to increase and then decreases with the increase of  $n_b$ . The critical values of  $\gamma$  for the onset of instability as a function of  $\log \beta$  are shown in Fig. 3.

To demonstrate the effects of Landau quantization on the EoS we show in Fig. 4 the EoS as a function of density with the field energy density  $B^2/8\pi$  subtracted. It is seen that the effects of Landau quantization are insignificant even at field values  $3 \times 10^{18}$  G and become sizable only for fields of order  $10^{19}$  G.

Here we should mention that the true minimum which determines the ground state of the matter is controlled by the Gibbs potential, which contains an additional term  $-\mathbf{M} \cdot \mathbf{B}$ . However, even at the strong field strength considered in the present study, the magnetization effect is negligible. For example, the contribution of the magnetization to  $P_{\perp}$  in Eq. (33) is

at least an order of magnitude smaller than that due to matter and field, depending upon the strength of the field and density. Hence, in our numerical results, we do not consider the effect of magnetization. However, it should be noted from Eqs. (33) and (34) that, in principle, the magnetization leads to anisotropy of the total pressure (apart from that due to the magnetic field itself) of the system and contributes negatively to the pressure in the perpendicular direction of the magnetic field. Therefore, inclusion of magnetization further strengthens the main conclusion of this study.

In this context, it is also worth mentioning that the effect of interaction of the anomalous magnetic moment (AMM) of particles with field could stiffen EoS [74]. It could, in principle, dominate over the softening effects by Landau quantization for fields strength above  $5 \times 10^{18}$  G *i.e.*, well above those considered in this paper. It will start dominating over the Landau quantization effect for fields  $\sim 5 \times 10^{19}$  G. However, at such a high field strength the field energy density contribution is more than enough to overwhelm the AMM interaction. Furthermore, the effect of AMM interaction is density dependent [74], and is most significant at low densities. In this respect, it is important to take a realistic profile of a star, which will suppress the contribution from AMM interactions because it implies low fields at low densities.

Finally, it is interesting to ask how the strong  $B$ -fields affect the composition of matter. In Fig. 5 we show the composition of hypernuclear matter as functions of matter density in cases with and without  $B$ -field. It is seen that the  $B$ -field has an effect only on the lightest particles, *i.e.*, the leptons. For these we see “Landau oscillations” in their population as a function of the field. These oscillations reflect the population of new Landau levels with the increase of the density. One should note that the field varies across the density profile between the two asymptotic values at the surface  $10^{15}$  G and the center of the star  $2 \times 10^{18}$  G. The matter at low densities experiences relatively low field, therefore the abundances in this region do not differ substantially from the free field case. It is only in the high density region that the field is large and appreciable deviations are seen for the leptons.

#### 4. Summary

We have studied the influence of strong magnetic fields on the high density nuclear matter, including the possible appearance of hyperons at large densities. In doing so we, first, have *implemented realistic density profiles of magnetic fields, which assume that the field decreases from the center of the star and reaches asymptotically its value at the surface  $\sim 10^{15}$  G characteristic for magnetars.* Thus, we have extended the previous studies of hypernuclear matter to realistic situation of density dependent field profiles and conducted a study of parameter space that defines the shape of the field profile.

Secondly, we have found that for sufficiently large fields  $B_c \geq 10^{18}$  G the matter becomes unstable. *The instability is associated with the negative contribution of the field pressure to the baryonic and leptonic pressures in the direction parallel to the magnetic field,* which renders the total pressure of the system anisotropic. We have found that the onset of instability depends on the magnetic field profile (parameterized in terms of  $\beta$  and  $\gamma$  parameters) as well as on the central field value  $B_c$ . The instability depends on particular values and form

of the density profile, but it sets in always for critical central field values  $B_{\text{cr}} \approx 10^{19}$  G for any values of  $\beta$  and  $\gamma$ . This gives a natural bound for the central magnetic field of neutron stars with *homogeneously distributed magnetic field*.

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## References

- [1] G. Vasisht, E. V. Gotthelf, The Discovery of an Anomalous X-Ray Pulsar in the Supernova Remnant Kes 73, *Astro. Phys. J.* 486 (1997) L129. [arXiv:astro-ph/9706058](#), [doi:10.1086/310843](#).
- [2] C. Kouveliotou, S. Dieters, T. Strohmayer, J. van Paradijs, G. J. Fishman, C. A. Meegan, K. Hurley, J. Kommers, I. Smith, D. Frail, T. Murakami, An X-ray pulsar with a superstrong magnetic field in the soft  $\gamma$ -ray repeater SGR1806 - 20, *Nature* 393 (1998) 235–237. [doi:10.1038/30410](#).
- [3] P. M. Woods, C. Kouveliotou, J. van Paradijs, K. Hurley, R. M. Kippen, M. H. Finger, M. S. Briggs, S. Dieters, G. J. Fishman, Discovery of a New Soft Gamma Repeater, SGR 1627-41, *Astro. Phys. J.* 519 (1999) L139–L142. [arXiv:astro-ph/9903267](#), [doi:10.1086/312124](#).
- [4] R. C. Duncan, C. Thompson, Formation of very strongly magnetized neutron stars - Implications for gamma-ray bursts, *Astro. Phys. J.* 392 (1992) L9–L13. [doi:10.1086/186413](#).
- [5] V. V. Usov, Millisecond pulsars with extremely strong magnetic fields as a cosmological source of gamma-ray bursts, *Nature* 357 (1992) 472–474. [doi:10.1038/357472a0](#).
- [6] C. Thompson, R. C. Duncan, The soft gamma repeaters as very strongly magnetized neutron stars - I. Radiative mechanism for outbursts, *Mon. Not. Roy. Astron. Soc.* 275 (1995) 255–300.
- [7] C. Thompson, R. C. Duncan, The Soft Gamma Repeaters as Very Strongly Magnetized Neutron Stars. II. Quiescent Neutrino, X-Ray, and Alfvén Wave Emission, *Astro. Phys. J.* 473 (1996) 322. [doi:10.1086/178147](#).
- [8] V. A. Ambartsumyan, G. S. Saakyan, The Degenerate Superdense Gas of Elementary Particles, *Astron. Zh.* 37 (1960) 193.
- [9] N. K. Glendenning, Neutron stars are giant hypernuclei?, *Astro. Phys. J.* 293 (1985) 470–493. [doi:10.1086/163253](#).
- [10] N. K. Glendenning, S. A. Moszkowski, Reconciliation of neutron-star masses and binding of the Lambda in hypernuclei, *Physical Review Letters* 67 (1991) 2414–2417. [doi:10.1103/PhysRevLett.67.2414](#).
- [11] F. Weber (Ed.), *Pulsars as astrophysical laboratories for nuclear and particle physics*, 1999.
- [12] A. Sedrakian, The physics of dense hadronic matter and compact stars, *Progress in Particle and Nuclear Physics* 58 (2007) 168–246. [arXiv:nucl-th/0601086](#), [doi:10.1016/j.ppnp.2006.02.002](#).
- [13] S. Balberg, I. Lichtenstadt, G. B. Cook, Roles of Hyperons in Neutron Stars, *Astro. Phys. J.* 121 (1999) 515–531. [arXiv:astro-ph/9810361](#), [doi:10.1086/313196](#).
- [14] H. Djapo, B.-J. Schaefer, J. Wambach, Appearance of hyperons in neutron stars, *Phys. Rev. C* 81 (3) (2010) 035803. [arXiv:0811.2939](#), [doi:10.1103/PhysRevC.81.035803](#).
- [15] M. Baldo, G. F. Burgio, H. . Schulze, Neutron Star Structure with Hyperons and Quarks, *ArXiv Astrophysics e-prints*[arXiv:astro-ph/0312446](#).
- [16] I. Vidaña, D. Logoteta, C. Providência, A. Polls, I. Bombaci, Estimation of the effect of hyperonic three-body forces on the maximum mass of neutron stars, *EPL (Europhysics Letters)* 94 (2011) 11002. [arXiv:1006.5660](#), [doi:10.1209/0295-5075/94/11002](#).
- [17] L. Bonanno, A. Sedrakian, Composition and stability of hybrid stars with hyperons and quark color-superconductivity, *Astron. and Astrophys.* 539 (2012) A16. [arXiv:1108.0559](#), [doi:10.1051/0004-6361/201117832](#).

- [18] W.-Z. Jiang, B.-A. Li, L.-W. Chen, Large-mass Neutron Stars with Hyperonization, *Astrophys. J.* 756 (2012) 56. [arXiv:1207.1686](#), [doi:10.1088/0004-637X/756/1/56](#).
- [19] I. Bednarek, P. Haensel, J. L. Zdunik, M. Bejger, R. Mańka, Hyperons in neutron-star cores and a  $2 M_{sun}$  pulsar, *Astron. and Astrophys.* 543 (2012) A157. [arXiv:1111.6942](#), [doi:10.1051/0004-6361/201118560](#).
- [20] M. Oertel, A. F. Fantina, J. Novak, Extended equation of state for core-collapse simulations, *Phys. Rev. C* 85 (5) (2012) 055806. [arXiv:1202.2679](#), [doi:10.1103/PhysRevC.85.055806](#).
- [21] K. Masuda, T. Hatsuda, T. Takatsuka, Hadron-Quark Crossover and Massive Hybrid Stars with Strangeness, *ArXiv e-prints*[arXiv:1205.3621](#).
- [22] R. Lastowiecki, D. Blaschke, H. Grigorian, S. Typel, Strangeness in the cores of neutron stars, *ArXiv e-prints*[arXiv:1112.6430](#).
- [23] S. Schramm, R. Negreiros, J. Steinheimer, T. Schürhoff, V. Dexheimer, Properties and Stability of Hybrid Stars, *ArXiv e-prints*[arXiv:1112.1853](#).
- [24] V. Dexheimer, J. Steinheimer, R. Negreiros, S. Schramm, Hybrid Stars in an SU(3) Parity Doublet Model, *ArXiv e-prints*[arXiv:1206.3086](#).
- [25] S. Weissenborn, D. Chatterjee, J. Schaffner-Bielich, Hyperons and massive neutron stars: The role of hyperon potentials, *Nuclear Physics A* 881 (2012) 62–77. [arXiv:1111.6049](#), [doi:10.1016/j.nuclphysa.2012.02.012](#).
- [26] S. Weissenborn, D. Chatterjee, J. Schaffner-Bielich, Hyperons and massive neutron stars: Vector repulsion and SU(3) symmetry, *Phys. Rev. C* 85 (6) (2012) 065802. [arXiv:1112.0234](#), [doi:10.1103/PhysRevC.85.065802](#).
- [27] F. Gulminelli, A. R. Raduta, J. Margueron, P. Papakonstantinou, M. Oertel, Phase diagram of neutron-rich nuclear matter and its impact on astrophysics, *ArXiv e-prints*[arXiv:1209.0270](#).
- [28] R. Mallick, Maximum mass of a hybrid star having a mixed phase region in the light of pulsar PSR J1614-2230, *ArXiv e-prints*[arXiv:1207.4872](#).
- [29] V. Canuto, J. Ventura, Quantizing Magnetic Fields in Astrophysics, *Fundam. Cosmic Phys.* 2 (1977) 203–353.
- [30] I. Fushiki, E. H. Gudmundsson, C. J. Pethick, Surface structure of neutron stars with high magnetic fields, *Astro. Phys. J.* 342 (1989) 958–975. [doi:10.1086/167653](#).
- [31] A. M. Abrahams, S. L. Shapiro, Equation of state in a strong magnetic field - Finite temperature and gradient corrections, *Astro. Phys. J.* 374 (1991) 652–667. [doi:10.1086/170151](#).
- [32] I. Fushiki, E. H. Gudmundsson, C. J. Pethick, J. Yngvason, Matter in a magnetic field in the Thomas-Fermi and related theories, *Annals of Physics* 216 (1992) 29–72. [doi:10.1016/0003-4916\(52\)90041-9](#).
- [33] O. E. Roegvaldsson, I. Fushiki, E. H. Gudmundsson, C. J. Pethick, J. Yngvason, Thomas-Fermi Calculations of Atoms and Matter in Magnetic Neutron Stars: Effects of Higher Landau Bands, *Astro. Phys. J.* 416 (1993) 276. [doi:10.1086/173234](#).
- [34] S. Chakrabarty, D. Bandyopadhyay, S. Pal, Dense Nuclear Matter in a Strong Magnetic Field, *Physical Review Letters* 78 (1997) 2898–2901. [arXiv:astro-ph/9703034](#), [doi:10.1103/PhysRevLett.78.2898](#).
- [35] D. Bandyopadhyay, S. Chakrabarty, P. Dey, S. Pal, Rapid cooling of magnetized neutron stars, *Phys. Rev. D* 58 (12) (1998) 121301. [arXiv:astro-ph/9804145](#), [doi:10.1103/PhysRevD.58.121301](#).
- [36] A. Broderick, M. Prakash, J. M. Lattimer, The Equation of State of Neutron Star Matter in Strong Magnetic Fields, *Astro. Phys. J.* 537 (2000) 351–367. [arXiv:astro-ph/0001537](#), [doi:10.1086/309010](#).
- [37] I.-S. Suh, G. J. Mathews, Cold Ideal Equation of State for Strongly Magnetized Neutron Star Matter: Effects on Muon Production and Pion Condensation, *Astro. Phys. J.* 546 (2001) 1126–1136. [arXiv:astro-ph/9912301](#), [doi:10.1086/318277](#).
- [38] A. K. Harding, D. Lai, Physics of strongly magnetized neutron stars, *Reports on Progress in Physics* 69 (2006) 2631–2708. [arXiv:astro-ph/0606674](#), [doi:10.1088/0034-4885/69/9/R03](#).
- [39] W. Chen, P.-Q. Zhang, L.-G. Liu, The Influence of the Magnetic Field on the Properties of Neutron Star Matter, *Modern Physics Letters A* 22 (2007) 623–629. [arXiv:astro-ph/0505113](#), [doi:10.1142/S0217732307023213](#).
- [40] A. Rabhi, C. Providência, J. Da Providência, Stellar matter with a strong magnetic field within



- density-dependent relativistic models, *Journal of Physics G Nuclear Physics* 35 (12) (2008) 125201. [arXiv:0810.3390](#), [doi:10.1088/0954-3899/35/12/125201](#).
- [41] M. Bocquet, S. Bonazzola, E.ourgoulhon, J. Novak, Rotating neutron star models with a magnetic field., *Astron. Astrophysics* 301 (1995) 757. [arXiv:gr-qc/9503044](#).
- [42] C. Y. Cardall, M. Prakash, J. M. Lattimer, Effects of Strong Magnetic Fields on Neutron Star Structure, *Astro. Phys. J.* 554 (2001) 322–339. [arXiv:astro-ph/0011148](#), [doi:10.1086/321370](#).
- [43] J. Frieben, L. Rezzolla, Equilibrium models of relativistic stars with a toroidal magnetic field, *ArXiv e-prints*[arXiv:1207.4035](#).
- [44] U. Das, B. Mukhopadhyay, Strongly magnetized cold degenerate electron gas: Mass-radius relation of the magnetized white dwarf, *Phys. Rev. D* 86 (4) (2012) 042001. [arXiv:1204.1262](#), [doi:10.1103/PhysRevD.86.042001](#).
- [45] U. Das, B. Mukhopadhyay, Violation of Chandrasekhar Mass Limit: the Exciting Potential of Strongly Magnetized White Dwarfs, *International Journal of Modern Physics D* 21 (2012) 42001. [arXiv:1205.3160](#), [doi:10.1142/S0218271812420011](#).
- [46] A. Kundu, B. Mukhopadhyay, Mass of Highly Magnetized White Dwarfs Exceeding the Chandrasekhar Limit: AN Analytical View, *Modern Physics Letters A* 27 (2012) 50084. [arXiv:1204.1463](#), [doi:10.1142/S0217732312500848](#).
- [47] R. A. Scalzo, G. Aldering, P. Antilogus, S. Aragon, C. Bailey, C. Baltay, S. Bongard, M. Buton, C. Childress, N. Chotard, Y. Copin, H. K. Fakhouri, A. Gal-Yam, E. Gangler, S. Hoyer, M. Kasliwal, S. Loken, P. Nugent, R. Pain, E. Pécontal, R. Pereira, S. Perlmutter, D. Rabinowitz, A. Rau, G. Rigaudier, K. Runge, G. Smadja, C. Tao, R. C. Thomas, B. Weaver, C. Wu, Nearby Supernova Factory Observations of SN 2007if: First Total Mass Measurement of a Super-Chandrasekhar-Mass Progenitor, *Astro. Phys. J.* 713 (2010) 1073–1094. [arXiv:1003.2217](#), [doi:10.1088/0004-637X/713/2/1073](#).
- [48] S. Chandrasekhar, E. Fermi, Problems of Gravitational Stability in the Presence of a Magnetic Field., *Astro. Phys. J.* 118 (1953) 116. [doi:10.1086/145732](#).
- [49] D. Lai, S. L. Shapiro, Cold equation of state in a strong magnetic field - Effects of inverse beta-decay, *Astro. Phys. J.* 383 (1991) 745–751. [doi:10.1086/170831](#).
- [50] M. Chaichian, S. S. Masood, C. Montonen, A. Pérez Martínez, H. Pérez Rojas, Quantum Magnetic Collapse, *Physical Review Letters* 84 (2000) 5261–5264. [arXiv:hep-ph/9911218](#), [doi:10.1103/PhysRevLett.84.5261](#).
- [51] A. Pérez Martínez, H. Pérez Rojas, H. J. Mosquera Cuesta, Anisotropic Pressures in Very Dense Magnetized Matter, *International Journal of Modern Physics D* 17 (2008) 2107–2123. [arXiv:0711.0975](#), [doi:10.1142/S0218271808013741](#).
- [52] X.-G. Huang, M. Huang, D. H. Rischke, A. Sedrakian, Anisotropic hydrodynamics, bulk viscosities, and r-modes of strange quark stars with strong magnetic fields, *Phys. Rev. D* 81 (4) (2010) 045015. [arXiv:0910.3633](#), [doi:10.1103/PhysRevD.81.045015](#).
- [53] X.-G. Huang, A. Sedrakian, D. H. Rischke, Kubo formulas for relativistic fluids in strong magnetic fields, *Annals of Physics* 326 (2011) 3075–3094. [arXiv:1108.0602](#), [doi:10.1016/j.aop.2011.08.001](#).
- [54] E. J. Ferrer, V. de La Incera, J. P. Keith, I. Portillo, P. L. Springsteen, Equation of state of a dense and magnetized fermion system, *Phys. Rev. C* 82 (6) (2010) 065802. [arXiv:1009.3521](#), [doi:10.1103/PhysRevC.82.065802](#).
- [55] L. Paulucci, E. J. Ferrer, V. de La Incera, J. E. Horvath, Equation of state for the magnetic-color-flavor-locked phase and its implications for compact star models, *Phys. Rev. D* 83 (4) (2011) 043009. [arXiv:1010.3041](#), [doi:10.1103/PhysRevD.83.043009](#).
- [56] V. Dexheimer, R. Negreiros, S. Schramm, Quark Deconfinement Under the Influence of Strong Magnetic Fields, *ArXiv e-prints*[arXiv:1108.4479](#).
- [57] A. Y. Potekhin, D. G. Yakovlev, Comment on “equation of state of a dense and magnetized fermion system”, *Phys. Rev. C* 85 (2012) 039801. [doi:10.1103/PhysRevC.85.039801](#).  
URL <http://link.aps.org/doi/10.1103/PhysRevC.85.039801>
- [58] E. J. Ferrer, V. de La Incera, J. P. Keith, I. Portillo, P. L. Springsteen, Reply to “Comment on ‘Equation of state of a dense and magnetized fermion system’ ”, *Phys. Rev. C* 85 (3) (2012) 039802.



- arXiv:1110.0420, doi:10.1103/PhysRevC.85.039802.
- [59] D. Bandyopadhyay, S. Chakrabarty, S. Pal, Quantizing Magnetic Field and Quark-Hadron Phase Transition in a Neutron Star, *Physical Review Letters* 79 (1997) 2176–2179. arXiv:astro-ph/9703066, doi:10.1103/PhysRevLett.79.2176.
- [60] J. D. Walecka, A theory of highly condensed matter., *Annals of Physics* 83 (1974) 491–529. doi:10.1016/0003-4916(74)90208-5.
- [61] J. Boguta, A. R. Bodmer, Relativistic calculation of nuclear matter and the nuclear surface, *Nuclear Physics A* 292 (1977) 413–428. doi:10.1016/0375-9474(77)90626-1.
- [62] N. K. Glendenning, The hyperon composition of neutron stars, *Physics Letters B* 114 (1982) 392–396. doi:10.1016/0370-2693(82)90078-8.
- [63] N. K. Glendenning, Hyperons in neutron stars., *Zeitschrift fur Physik A Hadrons and Nuclei* 326 (1987) 57–64.
- [64] N. K. Glendenning, Role of hyperons and pions in neutron stars and supernova, *Zeitschrift fur Physik A Hadrons and Nuclei* 327 (1987) 295–300. doi:10.1007/BF01284453.
- [65] S. A. Chin, A relativistic many-body theory of high density matter., *Annals of Physics* 108 (1977) 301–306. doi:10.1016/0003-4916(77)90016-1.
- [66] B. D. Serot, J. D. Walecka, The Relativistic Nuclear Many Body Problem, *Adv.Nucl.Phys.* 16 (1986) 1–327.
- [67] V. R. Khalilov, Macroscopic effects in cold magnetized nucleons and electrons with anomalous magnetic moments, *Phys. Rev. D* 65 (5) (2002) 056001. doi:10.1103/PhysRevD.65.056001.
- [68] C. Dover, A. Gal, SIGMA HYPERNUCLEI, *Comments Nucl.Part.Phys.* 12 (1984) 155–165. doi:10.1016/0375-9474(85)90509-3.
- [69] J. Schaffner, C. B. Dover, A. Gal, C. Greiner, D. J. Millener, H. Stocker, Multiply Strange Nuclear Systems, *Annals of Physics* 235 (1994) 35–76. doi:10.1006/aphy.1994.1090.
- [70] J. Schaffner, I. N. Mishustin, Hyperon-rich matter in neutron stars, *Phys. Rev. C* 53 (1996) 1416–1429. arXiv:nucl-th/9506011, doi:10.1103/PhysRevC.53.1416.
- [71] T. Fukuda, A. Higashi, Y. Matsuyama, C. Nagoshi, J. Nakano, M. Sekimoto, P. Tlustý, J. K. Ahn, H. En'yo, H. Funahashi, Y. Goto, M. Inuma, K. Imai, Y. Itow, S. Makino, A. Masaike, Y. Matsuda, S. Mihara, N. Saito, R. Susukita, S. Yokkaichi, K. Yoshida, M. Yoshida, S. Yamashita, R. Takashima, F. Takeuchi, S. Aoki, M. Ieiri, T. Iijima, T. Yoshida, I. Nomura, T. Motoba, Y. M. Shin, S. Weibe, M. S. Chung, I. S. Park, K. S. Sim, K. S. Chung, J. M. Lee, Cascade hypernuclei in the  $(K^-, K^+)$  reaction on  $^{12}\text{C}$ , *Phys. Rev. C* 58 (1998) 1306–1309. doi:10.1103/PhysRevC.58.1306.
- [72] P. Khaustov, D. E. Alburger, P. D. Barnes, B. Bassalleck, A. R. Berdoz, A. Biglan, T. Bürger, D. S. Carman, R. E. Chrien, C. A. Davis, H. Fischer, G. B. Franklin, J. Franz, L. Gan, A. Ichikawa, T. Iijima, K. Imai, Y. Kondo, P. Koran, M. Landry, L. Lee, J. Lowe, R. Magahiz, M. May, R. McCrady, C. A. Meyer, F. Merrill, T. Motoba, S. A. Page, K. Paschke, P. H. Pile, B. Quinn, W. D. Ramsay, A. Rusek, R. Sawafta, H. Schmitt, R. A. Schumacher, R. W. Stotzer, R. Sutter, F. Takeuchi, W. T. van Oers, K. Yamamoto, Y. Yamamoto, M. Yosoi, V. J. Zeps, Evidence of  $\Xi$  hypernuclear production in the  $^{12}\text{C}(K^-, K^+)^{12}\text{Be}$  reaction, *Phys. Rev. C* 61 (5) (2000) 054603. arXiv:nucl-ex/9912007, doi:10.1103/PhysRevC.61.054603.
- [73] S. Bart, R. E. Chrien, W. A. Franklin, T. Fukuda, R. S. Hayano, K. Hicks, E. V. Hungerford, R. Michael, T. Miyachi, T. Nagae, J. Nakano, W. Naing, K. Omata, R. Sawafta, Y. Shimizu, L. Tang, S. W. Wissink,  $\Sigma$  Hyperons in the Nucleus, *Physical Review Letters* 83 (1999) 5238–5241. doi:10.1103/PhysRevLett.83.5238.
- [74] A. E. Broderick, M. Prakash, J. M. Lattimer, Effects of strong magnetic fields in strange baryonic matter, *Physics Letters B* 531 (2002) 167–174. arXiv:astro-ph/0111516, doi:10.1016/S0370-2693(01)01514-3.